

# Parity Nonconservation in Strong Interactions

Vernon Barger,<sup>1</sup> Wai-Yee Keung,<sup>2</sup> and Chiu-Tien Yu<sup>1</sup>

<sup>1</sup>*Department of Physics, University of Wisconsin, Madison, WI 53706 USA*

<sup>2</sup>*Department of Physics, University of Illinois, Chicago, IL 60607-7059 USA*

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For top-quarks produced via the subprocess  $q\bar{q} \rightarrow t\bar{t}$ , the longitudinal t-quark polarization ( $P_{\parallel}$ ) vanishes in QCD.  $P_{\parallel}$  can be measured by the angular distribution of the lepton in  $t$ -quark semileptonic decay. New physics contributions that are parity nonconserving will be manifest by non-vanishing  $P_{\parallel}$ , which may be large. We illustrate this with the  $s$ -channel exchange of a massive  $X$ -gluon with chiral quark couplings.

Quantum Chromodynamics (QCD), the gauge field theory describing the strong interactions of colored quarks and gluons in the Standard Model<sup>1</sup>, has been extraordinarily successful in describing physics in both non-perturbative and perturbative regimes. Using the positivity of the Euclidean path integrand for Yang-Mills theory, Vafa and Witten proved that QCD does not spontaneously break parity or CP if  $\bar{\theta} = 0^2$ . CP conservation in the strong interactions is necessitated by the extreme smallness of experimental upper bounds on the neutron electric dipole moment<sup>3</sup>. It has been suggested that heavy ion collisions may form metastable phases which allow for parity nonconservation<sup>4</sup>. An induced charge separation along the angular momentum vector of the collision would create an electric dipole moment of the hot gluon matter. There are ongoing searches by the STAR collaboration<sup>5</sup> at the Relativistic Heavy Ion Collider (RHIC) to establish such an effect. Parity-violating effects can also be induced by topological solutions in QCD.

There have been many new physics models proposed to explain the large forward-backward asymmetry,  $A_{FB}$ , in top quark pair production seen at the Tevatron<sup>6</sup>. For example, recent works have shown that an axial gluon<sup>8,9,10,11</sup> can provide an explanation for the  $A_{FB}$  measurement. For recent reviews of the many new physics models, see e.g. Ref.<sup>8,12</sup>. However,  $A_{FB}$  is a test of charge-conjugation (at tree-level) and not of parity conservation. Instead, one can look at the longitudinal polarization of the top-quark, which is a quantity solely determined by parity nonconservation that can be measured in collider experiments. A model that can lead to observable parity nonconservation is the  $s$ -channel exchange of a spin-1  $X$ -gluon with both vector and axial-vector couplings to quarks<sup>9,13</sup>, which we will use as an illustrative example in this Letter. The importance of the measurements of the longitudinal top-quark polarization has also been noted by other authors<sup>14,15</sup>. At all orders in perturbation theory, QCD leads to zero longitudinal polarization, and SM

electroweak contributions should at most be at the few percent level. Thus, the longitudinal top polarization is free of QCD theory ambiguities, unlike the case for the forward-backward asymmetry or the transverse component of the top polarization, both of which have QCD contributions.

**$X$ -GLUON MODEL** Let  $A_1$  and  $A_2$  be non-abelian gauge fields associated with the gauge group product,  $SU_1(3) \times SU_2(3)$ . The full symmetry is broken by a bi-fundamental Higgs field  $\Phi$  with a vev of the form  $\langle \Phi \rangle = V\mathbf{1}$ . The surviving gauge symmetry is the vectorial  $SU_V(3)$ . Since  $T_2|0\rangle = -T_1|0\rangle$ , when the generators act upon the vev state, the massive  $X$ -gluon composition is

$$X = (g_1 A_1 - g_2 A_2) / \sqrt{g_1^2 + g_2^2}, \quad (1)$$

which has been normalized. The other orthogonal combination is the unbroken massless gluon field,

$$G = (g_2 A_1 + g_1 A_2) / \sqrt{g_1^2 + g_2^2}. \quad (2)$$

$X$ -gluon couplings to quarks The couplings to the generators are

$$g_1 A_1 T_1 + g_2 A_2 T_2 = \frac{1}{2} g_1 g_2 / g_X G (T_1 + T_2) + \frac{1}{2} g_X X (g_1^2 T_1 - g_2^2 T_2). \quad (3)$$

A further simplification gives

$$g_s G (T_1 + T_2) + \frac{1}{4} (g_1^2 - g_2^2) / g_X X (T_1 + T_2) + g_X X (T_1 - T_2). \quad (4)$$

where we define  $g_X = \frac{1}{2} \sqrt{g_1^2 + g_2^2}$  and  $g_s = \frac{1}{2} g_1 g_2 / g_X$ . We set  $T_1$  to act on  $L$  chiral fields and  $T_2$  on  $R$  such that  $T_1 + T_2$  acts only on the vectorial current, and  $T_1 - T_2$  on the axial-vectorial current:  $\mathbf{T}_1 + \mathbf{T}_2 \rightarrow \bar{q} \mathbf{T} \gamma^\mu q$ ,  $\mathbf{T}_1 - \mathbf{T}_2 \rightarrow -\bar{q} \mathbf{T} \gamma^\mu \gamma_5 q$ . The  $X$ -gluon interaction Lagrangian is

$$\mathbf{X} \cdot \bar{q} \mathbf{T} \gamma^\mu (g_V^q + g_A^q \gamma_5) q \quad (5)$$

with  $t \in q$  and  $g_V^2 = g_A^2 - g_s^2$ . This relationship of the couplings is modified if one considers higher dimension operators<sup>10</sup>

$$\begin{aligned} \mathcal{L} \supset & \Lambda^{-2} [\lambda_Q^2 (\bar{Q}_L \Phi) i \not{D} (\phi^\dagger Q) + \lambda_U^2 (\bar{U}_R \Phi^\dagger) i \not{D} (\phi U_R) \\ & + \lambda_D^2 (\bar{D}_R \Phi^\dagger) i \not{D} (\phi D_R)] \end{aligned} \quad (6)$$

<sup>a</sup> There are terms in the QCD Lagrangian that violate the charge-parity (CP) symmetry. Mechanisms have been proposed to solve this strong CP problem, of which the Peccei-Quinn mechanism is the most compelling<sup>3</sup>.

The vev of the bi-fundamental Higgs  $\phi$  allows the left-handed gauge field to act upon the right handed quark, and vice versa such that  $A_1$  acts on  $\mathbf{T}_1 + y_{U|D}\mathbf{T}_2$  and  $A_2$  acts on  $\mathbf{T}_2 + x\mathbf{T}_1$ , where  $x = \lambda_Q^2 V^2/\Lambda^2$  and  $y_{U|D} = \lambda_{U|D}^2 V^2/\Lambda^2$ . We also have

$$\mathbf{T}_1 \pm \mathbf{T}_2 \Rightarrow (1 \pm x)\mathbf{T}_1 \pm (1 \pm y)\mathbf{T}_2. \quad (7)$$

The kinetic derivative piece  $i\partial$  is increased by  $1+x$  for  $Q$  and  $1+y_{U|D}$  for  $U$  or  $D$  respectively. After renormalizing the kinetic pieces, we have

$$\begin{aligned} g_A^q &= -\frac{g_X}{2} \left( \frac{1-x}{1+x} + \frac{1-y_q}{1+y_q} \right), \\ g_V^q &= \frac{g_1^2 - g_2^2}{4g_X} + \frac{g_X}{2} \left( \frac{1-x}{1+x} - \frac{1-y_q}{1+y_q} \right). \end{aligned} \quad (8)$$

The restrictions on the couplings noted above disappear when higher dimension operators are included.

$q\bar{q} \rightarrow t\bar{t}$  The helicity amplitudes for the subprocess  $q\bar{q} \rightarrow t\bar{t}$ , shown in Fig.1, are given in Table I, where  $\theta$  is the CM scattering angle,  $\beta^2 = 1 - 4m_t^2/\hat{s}$ , and

$$\begin{aligned} G_{I,V} &= \frac{g_s^2}{\hat{s}} + \frac{g_I^q g_V^t}{\hat{s} - m_X^2 + im_X \Gamma_X}, \\ G_{I,A} &= \frac{g_I^q g_A^t}{\hat{s} - m_X^2 + im_X \Gamma_X} \end{aligned} \quad (9)$$

The  $G_{I,V(A)}$  are functions of  $\hat{s}$ , the square of the subprocess center-of-mass energy, and carry two subscripts; The first refers to initial quark chiralities,  $I = L$  or  $R$  where we define  $g_L = \frac{1}{2}(g_V - g_A)$ ,  $g_R = \frac{1}{2}(g_V + g_A)$ . The massless condition on initial quarks simplifies the calculation with couplings in this basis. The second subscript refers to the vectorial or axial-vectorial nature of the top quark couplings, which is more efficient in dealing with massive states.

We define  $\tilde{\sigma}(\theta) = \sum |\mathcal{M}|^2(q\bar{q} \rightarrow t\bar{t})$ , which is the subprocess differential cross-section modulo an overall factor<sup>b</sup>. Then, we have

$$\begin{aligned} \tilde{\sigma}(\theta) &= [A^+(-\beta) + A^+(\beta)](1 + \cos^2 \theta) \\ &\quad - 2[A^-(\beta) - A^-(-\beta)] \cos \theta \\ &\quad + 2(1 - \beta^2)A^+(0) \sin^2 \theta \end{aligned} \quad (10)$$

where

$$A^\pm(\beta) = \hat{s}^2 (|G_{L,V} + \beta G_{L,A}|^2 \pm |G_{R,V} + \beta G_{R,A}|^2). \quad (11)$$

Note that the second line of Eq.10 gives rise to the forward-backward asymmetry. Since the  $s$ -channel gluon and  $X$ -gluon amplitudes have identical color structure, the polarization and asymmetry predictions are independent of the parton distribution functions.

For the 7 TeV run of the LHC (LHC7), the analysis is complicated by subprocesses that involve gluons as partons. By a selective choice of the rapidity region that emphasizes the  $q\bar{q} \rightarrow t\bar{t}$  subprocess, it may be possible to probe parity and  $C$  nonconservation at the LHC<sup>15</sup>, as well as at the Tevatron.

### LONGITUDINAL POLARIZATION OF TOP

For our purposes, we will consider the leading-order production of top quarks. To all orders of QCD, the top quarks produced are unpolarized. However, in  $X$ -gluon models, the chiral structure gives rise to partially polarized tops. The longitudinal polarization of the top is described by

$$P_{\parallel} = \frac{\sum [|(h_q, h_{\bar{q}}, +, h_{\bar{t}})|^2 - |(h_q, h_{\bar{q}}, -, h_{\bar{t}})|^2]}{\sum [|(h_q, h_{\bar{q}}, +, h_{\bar{t}})|^2 + |(h_q, h_{\bar{q}}, -, h_{\bar{t}})|^2]} = \frac{\tilde{\sigma}_{\parallel}(\theta)}{\tilde{\sigma}(\theta)} \quad (12)$$

where the sum is over helicities and  $\tilde{\sigma}(\theta)$  is given by Eq. 10. The numerator can be simplified as

$$\begin{aligned} \tilde{\sigma}_{\parallel}(\theta) &= [A^+(\beta) - A^+(-\beta)](1 + \cos^2 \theta) \\ &\quad - 2[A^-(-\beta) + A^-(\beta)] \cos \theta \end{aligned} \quad (13)$$

The antilepton  $\ell^+$  from the top decay has an angular distribution given by  $(1 + P_{\parallel} \cos \psi)$ , where  $\psi$  is defined in Eq.16. We obtain a similar expression for  $\bar{\ell}$  by exchanging  $h_t \leftrightarrow h_{\bar{t}}$ . The corresponding angular distribution of  $\ell^-$  is  $1 - \bar{P}_{\parallel} \cos \psi$ . From CP-nonconservation,  $P_{\parallel} = -\bar{P}_{\parallel}$ , so that the angular distributions of  $\ell^\pm$  are symmetric under CP.

The angle  $\psi$  is defined as the angle between the  $\ell^+$  and the negative  $\bar{t}$  momentum in a boosted frame in which the  $t$  is at rest. The Lorentz boost to the  $t\bar{t}$  CM frame gives

$$E_{t\bar{t}}(e^+) = E_t(e^+)(1 + \beta \cos \psi) \left( \frac{1}{2} M_{t\bar{t}}/m_t \right) \quad (14)$$

so that

$$(2m_t/M_{t\bar{t}})p_{\ell^+} \cdot (p_t + p_{\bar{t}})/M_{t\bar{t}} = p_{\ell^+} \cdot p_t(1 + \beta \cos \psi)/m_t \quad (15)$$

$$\cos \psi = \left[ \frac{2m_t^2}{M_{t\bar{t}}^2} \left( \frac{p_{\ell^+} \cdot p_{\bar{t}}}{p_{\ell^+} \cdot p_t} + 1 \right) - 1 \right] / \sqrt{1 - \frac{4m_t^2}{M_{t\bar{t}}^2}} \quad (16)$$

The expression above makes use of covariant 4-dot-products and can be evaluated in any frame. Here, we assume all momenta can be reconstructed in the experiment.  $\bar{\psi}$  is obtained from Eq.16 with substitutions  $\psi \rightarrow \bar{\psi}$ ,  $\ell^+ \rightarrow \ell^-$ , and  $t \leftrightarrow \bar{t}$ .

**PHENOMENOLOGY** For our illustrations, we adopt the parameters  $|g_A| = g_s/3$ ,  $M_X = 420$  GeV, and  $\Gamma_X = 42$  GeV of the  $A_{FB}$  model of Ref.<sup>10</sup>, but we allow for the possibility of a vector coupling as well, which leads to parity nonconservation. For simplicity, we consider maximal parity nonconservation scenarios, which we denote as  $V \pm A$ . The new physics contribution to the  $M_{t\bar{t}}$  distribution is found to be similar in the  $V \pm A$  cases to that for  $A$ -only in Ref.<sup>10</sup>, as shown in Fig.2. We do

<sup>b</sup>  $\frac{d\tilde{\sigma}}{d\cos\theta} = \frac{\beta}{576\pi\hat{s}} \tilde{\sigma}(\theta)$ .

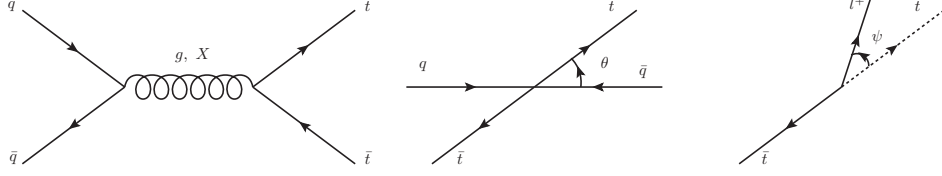


FIG. 1: Feynman diagram of the  $s$ -channel exchanges of the gluon and  $X$ -gluon in the  $q\bar{q} \rightarrow t\bar{t}$  subprocess and the definitions of  $\theta$  and  $\psi$ . The dotted line for  $t$  denotes a boost into the  $t$  rest frame.

TABLE I: Helicity amplitudes for  $q\bar{q} \rightarrow t\bar{t}$

Final State Polarizations	Initial State Polarizations	
	$-+$	$+-$
$--$	$G_{L,V} 2\sqrt{s} m_t \sin \theta$	$G_{R,V} 2\sqrt{s} m_t \sin \theta$
$++$	$-G_{L,V} 2\sqrt{s} m_t \sin \theta$	$-G_{R,V} 2\sqrt{s} m_t \sin \theta$
$-+$	$-(G_{L,V} - \beta G_{L,A}) \hat{s}(1 + \cos \theta)$	$(G_{R,V} - \beta G_{R,A}) \hat{s}(1 - \cos \theta)$
$+-$	$(G_{L,V} + \beta G_{L,A}) \hat{s}(1 - \cos \theta)$	$-(G_{R,V} + \beta G_{R,A}) \hat{s}(1 + \cos \theta)$

not take into account smearing due to the experimental  $M_{t\bar{t}}$  resolution. We show the dependence of the  $A_{FB}$  on top-pair invariant mass  $M_{t\bar{t}}$  in Fig. 3(a).

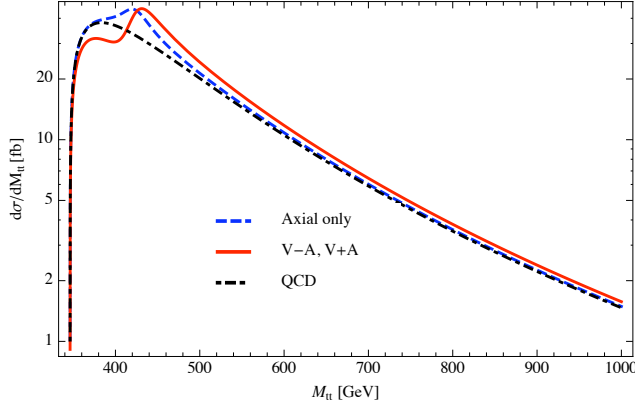


FIG. 2: The new physics (NP) contributions to  $d\sigma/dM_{t\bar{t}}$  vs.  $M_{t\bar{t}}$  in the  $X$ -gluon model at the Tevatron.

We show the dependence of  $P_{\parallel}$  on  $M_{t\bar{t}}$  in Fig. 3(b). The polarization goes to zero near  $M_{t\bar{t}} = M_X$ .  $P_{\parallel}$  is largest for  $\theta = \pi$ . Zero polarization is predicted for a purely axial or vector coupling.

**Tri-gauge Boson Couplings** The trilinear couplings in the  $X$ -gluon model are

$$f_{ijk} \langle g_s G^i G^j G^k + g_s G^i X^j X^k + 2g_X X^i X^j X^k \rangle \quad (17)$$

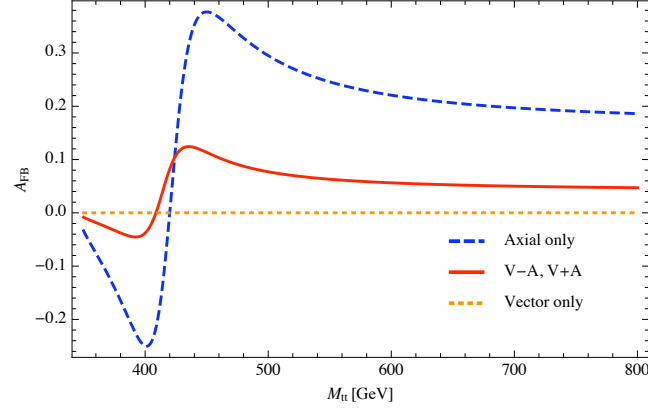
where the bra-ket denotes

$$\begin{aligned} \langle \phi^i \phi^j \phi^k \rangle = & \phi_\mu^i \phi_\nu^j \partial^\mu \phi^k{}^\nu + \phi_\mu^j \phi_\nu^k \partial^\mu \phi^i{}^\nu + \phi_\mu^k \phi_\nu^i \partial^\mu \phi^j{}^\nu \\ & - \phi_\mu^i \phi_\nu^k \partial^\mu \phi^j{}^\nu - \phi_\mu^j \phi_\nu^i \partial^\mu \phi^k{}^\nu - \phi_\mu^k \phi_\nu^j \partial^\mu \phi^i{}^\nu \end{aligned} \quad (18)$$

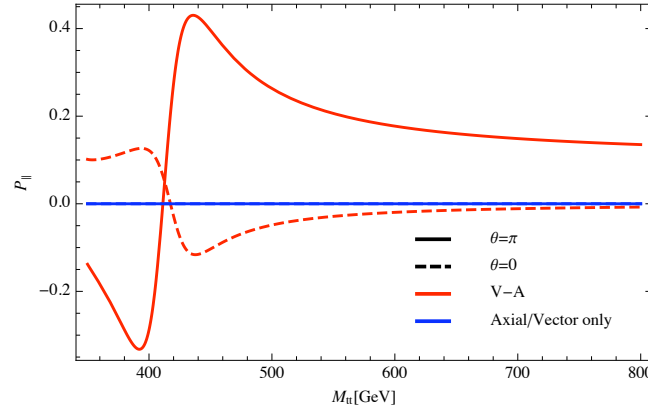
which gives antisymmetric dependence on momenta. There is also a 4-particle coupling. Thus, the  $X$ -gluon can be strongly pair-produced by gluon fusion via an  $s$ -channel gluon. Searches for the  $X$ -gluon are dependent on its decay modes, which are multijets in the scenario of Ref.<sup>10</sup>. It is necessary to measure the tri-gauge couplings of the  $X$ -gluon to prove its  $SU(3)$  color property.

**Conclusions** For top-quarks produced via the subprocess  $q\bar{q} \rightarrow t\bar{t}$ , the longitudinal  $t$ -quark polarization ( $P_{\parallel}$ ) vanishes in QCD. As a new physics illustration, we have shown that the  $s$ -channel exchange of a massive  $X$ -gluon with chiral quark couplings gives rise to a substantial  $P_{\parallel}$ . Our study emphasizes the low-energy phenomenology and its parity nonconservation. Additional fermions are needed for UV completion and to cancel anomalies. The longitudinal polarization is a measurement of  $\sigma_t \cdot (\mathbf{p}_t - \mathbf{p}_{\bar{t}})/|\mathbf{p}_t - \mathbf{p}_{\bar{t}}|$  in the  $t$  rest frame. Being odd in spatial parity, it is expected to be zero in all orders of perturbative QCD. Thus measurements of  $P_{\parallel}$  in  $t\bar{t}$  events arising from the  $q\bar{q} \rightarrow t\bar{t}$  subprocess at the Tevatron and LHC could prove to be of fundamental importance in finding parity-violation in strong interactions beyond QCD.

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(a)  $A_{FB}$  vs.  $M_{t\bar{t}}$  for  $V \pm A$  (solid), Axial-only (dashed), and Vector-only (dotted) couplings.



(b)  $P_{||}$  vs.  $M_{t\bar{t}}$  for  $\theta = 0$  (dashed) and  $\pi$  (solid).  $P_{||}$  for  $V + A$  is opposite in sign to  $P_{||}$  for  $V - A$ .

FIG. 3:  $A_{FB}$  and  $P_{||}$  vs.  $M_{t\bar{t}}$  for  $M_X = 420$  GeV,  $\Gamma_X = 42$  GeV, and  $|g_A| = g_s/3$ .

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